

CoCoALib - Feature #738

Extend homomorphism to polynomial ring

25 Jun 2015 14:41 - John Abbott

Status:	In Progress	Start date:	25 Jun 2015
Priority:	Normal	Due date:	
Assignee:		% Done:	10%
Category:	New Function	Estimated time:	3.00 hours
Target version:	CoCoALib-1.0	Spent time:	1.00 hour
Description			
It might be nice to have a simple/convenient/compact way of extending "coefficient homomorphisms" to polynomial rings. Currently one has to create a polyringhom, and this requires saying how the indets map (which decreases readability).			
Related issues:			
Related to CoCoALib - Feature #992: Poly ring homomorphism to change ordering		New	13 Dec 2016
Related to CoCoA-5 - Feature #7: Automatic mapping between (some) rings		Resolved	20 Oct 2011

History

#1 - 25 Jun 2015 14:48 - John Abbott

What precisely are the semantics? When can the function be called?
Various cases to consider:

1. given $\phi: R \rightarrow R$ extend to $\psi: R[x,y,z] \rightarrow R[x,y,z]$
2. given $\phi: R \rightarrow S$ extend to $\psi: R[x,y,z] \rightarrow S[x,y,z]$
3. given $\phi: R \rightarrow R$ extend to $\psi: R[x,y,z] \rightarrow R[a,b,c]$
4. given $\phi: R \rightarrow S$ extend to $\psi: R[x,y,z] \rightarrow S[a,b,c]$

We could also consider a codomain with more indets than the domain, but that is probably better handled explicitly by PolyRingHom.

#2 - 25 Jun 2015 15:28 - Anna Maria Bigatti

- Status changed from New to In Progress
- % Done changed from 0 to 10
- Estimated time set to 3.00 h

I don't like automatic choices: look at these examples

```
R[x_1, x_2] --> R[x_0, x_1, x_2]
R[x_1, x_2] --> R[x_0, x_1]
R[a, b] --> R[x, y, a, b]
```

Obviously the meaning depends on who is "thinking" this maps:

- 1 - if a user actually writes it then it probably means that he wants to preserve names
- 2 - if it is part of a program (creating a new ring) then it probably means i-th into i-th

For the "user" option we could make two functions PreserveNamesRingHom/PreserveNamesAlgebraHom.

#3 - 25 Jun 2015 15:48 - John Abbott

Here is the original situation where the problem arose. I Have some polynomials with complex coeffs (in $\mathbb{C}[i]$) and I want to define "complex conjugation" on $\mathbb{C}[i]$ and extend it to $\mathbb{C}[i][x]$. Currently it takes several steps to achieve this.

```
use QQI ::= QQ[I];
minpoly := ideal(I^2+1);
conj1 := PolyAlgebraHom(QQI,QQI,[-I]);
Qi := NewQuotientRing(QQI,minpoly);
conj2 := CanonicalHom(QQI,Qi)(conj1);
conj3 := InducedHom(Qi,conj2);
P ::= Qi[X[1..3]];
use P;
conj := PolyRingHom(P,P,conj3,indets(P));
```

#4 - 25 Jun 2015 17:23 - John Abbott

Anna, John and Renzo agree that cases (1) and (2) in comment 1 are OK, and that the others are best handled by constructing explicitly the homomorphism saying precisely where each indet should go.

To be more precise: the automatic mapping of indets is allowed only if they are identical: **same number, same names, and same order of appearance**.

Anna suggests that we do not require the term ordering to be the same; John notes that if the terms orderings are the same, the implementation could be both simple and quick, whereas allowing a change of ordering seems to be both more complex and slower (e.g. geobuckets). The more general implementation is probably more useful to the user.

#5 - 13 Dec 2016 18:12 - John Abbott

- Related to Feature #992: Poly ring homomorphism to change ordering added

#6 - 30 Nov 2017 10:02 - John Abbott

- Related to Feature #7: Automatic mapping between (some) rings added