CoCoALib - Design #377

IsDivisible -- exact semantics?

19 Jun 2013 10:28 - John Abbott

Status:	Closed	**Start date:**	19 Jun 2013
Priority:	Normal	**Due date:**	
Assignee:	John Abbott	**% Done:**	100%
Category:	Maths Bugs	**Estimated time:**	3.00 hours
Target version:	CoCoALib-0.99533 Easter14	**Spent time:**	3.00 hours

Description

While dealing with issue #248 I realised that the exact semantics of IsDivisible are not clear.
The problem is when the ring contains zero-divisors.

Guideline: IsDivisible(a,b) gives **true** iff a/b will succeed

Let zd be a (non-zero) zero-divisor; let nzd be a non-zero-divisor:
- it is clear that IsDivisible(0,nzd) should produce true
- what should IsDivisible(0,zero(R)) produce?
- what should IsDivisible(0,zd) produce?
- what should IsDivisible(2*zd,zd) produce?

For concreteness, you can take the ring R = NewZZmod(6) and zd = 2 and nzd = 5

Comments?

Related issues:

Related to CoCoALib - Feature #248: IsDivisible for RingElem with nice interface	**Closed**	**01 Oct 2012**
Related to CoCoALib - Design #1500: IsDivisible in a field?	**Closed**	**05 Oct 2020**

History

#1 - 19 Jun 2013 10:57 - John Abbott

- Status changed from New to In Progress

- Assignee set to John Abbott

The problem with dividing 4/2 in ZZ/6 is that the true answer is 2 in ZZ/3 -- a different ring! The answer could be either 2 or 5 in ZZ/6.

(A) So we could say that 4 **is divisible** by 2, but when we perform the division we must **choose one answer among many**.

(B) Or we could say that 4 **is not divisible** by 2 because the answer is **not unique** in that ring.

Consequences:

(C) If we adopt approach (A) then presumably we must also say that 0 is divisible by 0; but that implies that one can compute 0/0 and expect to get an answer...

(D) If we adopt approach (B) then IsDivisible(0,zd) should always give false (because the answer is not unique: it could be 0 or any *cofactor* of zd).

At the moment I favour approach (B).

#2 - 20 Jun 2013 14:46 - John Abbott

- % Done changed from 0 to 10

JAA continues to believe that attempting to compute 0/0 in any ring should give an error. Giving an answer is almost surely going to lead to a *nasty surprise* sooner or later.

This reinforces my preference for design decision **(B)**.

#3 - 29 Oct 2013 14:59 - Anna Maria Bigatti

- Target version changed from CoCoALib-0.99534 Seoul14 to CoCoALib-0.99532

#4 - 01 Apr 2014 17:29 - Anna Maria Bigatti

- Target version changed from CoCoALib-0.99532 to CoCoALib-0.99533 Easter14

#5 - 04 Apr 2014 00:31 - John Abbott

- % Done changed from 10 to 30

Summarising:
IsDivisible(a,b) gives true iff there is a **unique** c in the ring satisfying a = b*c (assuming ring is commutative). This implies that a/b is well-defined (and so ought to be computable).

As Anna said: this fits in well with CoCoA's "pragmatic philosphy".

Nevertheless the documentation should point out the pecularities of IsDivisible in CoCoA.

Note that IsDivisible throws ERR::DivByZero if b=0; we chose this behaviour because we think that testing for divisibility by 0 is more likely a consequence of a programming error than an intended test.

Note that IsDivisible apparently always gives false if the 2nd arg is a non-zero zero-divisor (agreeing with condition that the quotient be unique).

Action: check & correct documentation, check & correct implementations!

#6 - 08 Apr 2014 16:20 - John Abbott

- % Done changed from 30 to 50

Aldo says that "a is divisible by b" means that there exists at least one c such that a = b*c. He accepted happily that this means that 0 is divisible by 0; this is, of course, quite unacceptable for CoCoA (because 0/0 will cause an error; it'd be too "dangerous" to give a result).

After some discussion the proposal is:

- if b is a zero-divisor then give error (ERR::DivByZero)
- otherwise return true or false appropriately

I observe that in comment-5 we had proposed error for b=0 but not for other zero-divisors; this is somewhat inconsistent! The proposal in this comment is more consistent.

#7 - 08 Apr 2014 17:59 - John Abbott

- Status changed from In Progress to Feedback

- % Done changed from 50 to 90

Implemented the proposal in comment-6; changed several IsZero checks into IsZeroDivisor.

Changed state to feedback

#8 - 15 Apr 2014 15:50 - John Abbott

- Status changed from Feedback to Closed

- % Done changed from 90 to 100

#9 - 17 Apr 2014 09:21 - Anna Maria Bigatti

- Estimated time set to 3.00 h

#10 - 05 Oct 2020 14:33 - John Abbott

- Related to Design #1500: IsDivisible in a field? added