

## CoCoALib - Design #254

### How to return a Hilbert Series in CoCoALib

08 Oct 2012 16:00 - Anna Maria Bigatti

<b>Status:</b>	Closed	<b>Start date:</b>	08 Oct 2012
<b>Priority:</b>	Normal	<b>Due date:</b>	
<b>Assignee:</b>	John Abbott	<b>% Done:</b>	100%
<b>Category:</b>	New Function	<b>Estimated time:</b>	6.00 hours
<b>Target version:</b>	CoCoALib-0.99533 Easter14	<b>Spent time:</b>	5.10 hours
<b>Description</b>			
Right now Hilbert series do not exist in CoCoALib. In CoCoA-5 they are built in the package hp.cpkg5.  Now we need to clean it up at fix a common interface for the Hilbert Series in TmpHilbert and in Externallib-Normaliz.  We should make something like the <b>factorization</b> class and make a related <b>from</b> function for sending this class to CoCoA-5.			
<b>Related issues:</b>			
Related to CoCoALib - Feature #203: Function to get the Hilbert Polynomial ri...		<b>Closed</b>	<b>04 Jul 2012</b>
Related to CoCoA-5 - Feature #93: Hilbert series: global output rings for non...		<b>Closed</b>	<b>23 Feb 2012</b>
Related to CoCoA-5 - Design #376: Hilbert: Redesign hp.cpkg5 and public funct...		<b>In Progress</b>	<b>18 Jun 2013</b>
Related to CoCoALib - Feature #390: Store unique copy of QQ[t_1..t_n] (RingQQ...		<b>Closed</b>	<b>23 Jul 2013</b>
Related to CoCoALib - Feature #399: add myHilbertSeries member field to ideal...		<b>New</b>	<b>02 Aug 2013</b>
Related to CoCoA - Support #425: Osnabruock 2014-01		<b>Closed</b>	<b>27 Jan 2014</b>
Related to CoCoA-5 - Design #493: Cleanup Hilbert package hp.cpkg5		<b>Closed</b>	<b>24 Mar 2014</b>

### History

#### #1 - 12 Oct 2012 19:27 - Anna Maria Bigatti

- Tracker changed from Feature to Design
- Category set to New Function
- Target version set to CoCoALib-0.9953

#### #2 - 19 Mar 2013 10:35 - Christof Soeger

Any news on it?

In libnormaliz it is a vector for the coefficients of the numerator and a map for the exponents of  $(1-t^i)^e$ . However I do not suggest this as type in CoCoALib.

I think the type should offer the possibility to easily

- access the data, for example to check if the h-vector is palindromic,
- do arithmetic operations, like adding two series (as rational functions).

In connection to this we also need a type for Hilbert (quasi-)polynomials

I suggest to have a unique ring where all these polynomials could live in. For me a univariate polynomial ring over QQ is enough, but other applications could profit from such a multivariate ring.

Do you have any support for quasi-polynomials?

### #3 - 20 Mar 2013 23:17 - John Abbott

- Status changed from New to In Progress

Here is a suggestion: a RECORD with two fields numer and denom where

- numer is a polynomial,
- and denom is a RECORD with the same structure as would be returned by a call to factor (*i.e.* fields factors, multiplicities and RemainingFactor).

There is also the matter *to which ring do these polynomials belong?*

### #4 - 17 Jun 2013 11:59 - Anna Maria Bigatti

- % Done changed from 0 to 20

Design:

new class **HilbertSeries**: member fields  
**myNum** RingElem and **myDenFactors** factorization.

Constructors:

(RingElem num, factorization<RingElem> DenFactors) -- general  
(vector<BigInt> DenseRepr, vector<long> DenExponents) -- for univariate HS  
e.g. DenExps = [1,3,2,2,3] --> (1-t) \* (1-t^3)^2 \* (1-t^2)^2

Guarantees:

**HilbertSeriesPolyRing(long (or MachineInt?) n)**: ring for all num and den factors.  
No guarantees on num or den (irreducibility, order of factors, ...)

Operations:

constructors (copy ctor)  
assignment  
operator<<  
operator== (H1==H2 iff they are the same in FrFld(P))  
operator!=  
num(HS) (same as internally stored: no guarantees on simplification)  
DenFactors(HS)  
wish list: conversion into rational function (needs global FractionField)

### #5 - 17 Jun 2013 16:08 - Christof Soeger

I spoke with W.Bruns about a name for the ring. One suggestion is PolynomialInvarianceRing (or a shortened form) instead of HilbertSeriesPolyRing. We think it is a better description and allows a more flexible use.

In fact even in the Hilbert series and the Hilbert polynomial the meanings of the variables are different. For the Hilbert polynomial the variable is a degree in the monoid, algebra, ... and for the Hilbert series it is a auxiliary variable to form the series. Comparison between these to objects as elements in one polynomial ring is not useful.

But we don't think there should be two different rings.

**#6 - 18 Jun 2013 08:54 - Anna Maria Bigatti**

Christof Soeger wrote:

I spoke with W.Bruns about a name for the ring. One suggestion is PolynomialInvarianceRing (or a shortened form) instead of HilbertSeriesPolyRing. We think it is a better description and allows a more flexible use.

... what about **RingQQt(n)**? That is pretty self-explanatory and flexible ;-)

**#7 - 18 Jun 2013 14:15 - Christof Soeger**

Anna Maria Bigatti wrote:

... what about **RingQQt(n)**? That is pretty self-explanatory and flexible ;-)

Yes that is flexible :) A good possibility.

In CoCoA 5 we will also have this ring(s)?

**#8 - 18 Jun 2013 18:40 - John Abbott**

- % Done changed from 20 to 50

New name implemented & checked-in.

**#9 - 18 Jun 2013 19:12 - Anna Maria Bigatti**

- Assignee set to John Abbott

- % Done changed from 50 to 80

Christof Soeger wrote:

Anna Maria Bigatti wrote:

... what about **RingQQt(n)**? That is pretty self-explanatory and flexible ;-)

In CoCoA 5 we will also have this ring(s)?

Yes, checked in

**#10 - 23 Jul 2013 18:16 - Anna Maria Bigatti**

- Subject changed from *How to return a Hilbert Series* to *How to return a Hilbert Series in CoCoALib*

**#11 - 29 Oct 2013 13:39 - Anna Maria Bigatti**

- Target version changed from *CoCoALib-0.9953* to *CoCoALib-0.99532*

**#12 - 28 Jan 2014 18:04 - Christof Soeger**

- % Done changed from 80 to 90

Anna Maria Bigatti wrote:

```
Constructors:
(RingElem num, factorization<RingElem> DenFactors) -- general
(vector<BigInt> DenseRepr, vector<long> DenExponents) -- for univariate HS
e.g. DenExps = [1,3,2,2,3] --> (1-t) * (1-t^3)^2 * (1-t^2)^2
```

I implemented the constructor for univariate HS. It is used in the new function `Normaliz::HilbertSeries(cone c)`.

**#13 - 29 Jan 2014 19:50 - Anna Maria Bigatti**

- Status changed from *In Progress* to *Resolved*

Now all essentially set.

One thing to decide: factorization of denom is now arbitrary:

Normaliz uses a compact form (i.e.  $(1-t)^2(1-t^3)^4$ )

CoCoA uses the long form with all multiplicities == 1.

Should we set for a specific form?

Should we offer conversion from compact to/from long form?

Which functions should be available on class **HPSeries**?

Surely a **simplify**, to get HVector and dimension (with standard grading)

**#14 - 01 Apr 2014 17:28 - Anna Maria Bigatti**

- Target version changed from *CoCoALib-0.99532* to *CoCoALib-0.99533 Easter14*

**#15 - 17 Apr 2014 00:50 - John Abbott**

I'm not an expert in the field, and in my ignorance find the "compact" form more satisfactory. I could even envisage a mem fn for factorization which makes the factors compact (i.e. gathers together equal factors and sums their multiplicities).

It could be nice if the "compact" form had the factors in increasing order of degree (first 1-t then 1-t^2 and so on).

**#16 - 17 Apr 2014 09:24 - Anna Maria Bigatti**

- *Estimated time set to 6.00 h*

**#17 - 17 Apr 2014 15:19 - Anna Maria Bigatti**

- *Status changed from Resolved to Closed*

- *% Done changed from 90 to 100*

(for now) we have decided that factorization of denom is arbitrary.  
There will be functions to clean up a factorization.