## CoCoALib - Bug \#1641

## gcd does not recognize univariate input

20 Dec 2021 13:43 - John Abbott

| Status: | Closed | Start date: | 20 Dec 202 |  |
| :---: | :---: | :---: | :---: | :---: |
| Priority: | High | Due date: |  |  |
| Assignee: | John Abbott | \% Done: | 100\% |  |
| Category: | Improving | Estimated time: | 5.33 hours |  |
| Target version: | CoCoALib-0.99850 | Spent time: | 5.35 hours |  |
| Description <br> Determinants where you would not expect them! |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| As soon as there is more than 1 indet in the ring CoCoA "stupidly" uses the syzygy method to compute the gcd even if the input polys are univariate! |  |  |  |  |
| Improve! |  |  |  |  |
| Related issues: |  |  |  |  |
| Related to CoCoALib - Feature \#1197: IsZeroDet: new fn |  |  | In Progress | 26 Jun 2018 |
| Related to CoCoALib - Design \#1649: Add file SparsePolyOps-vector.C |  |  | Closed | 21 Jan 2022 |
| Related to CoCoALib - Slug \#1057: Slug: Polynomial ring contructor slow with ... |  |  | In Progress | 04 May 2017 |
| Related to CoCoA-5 - Slug \#1068: PolyRing constructor: NewOrdvArith computed ... |  |  | In Progress | 17 May 2017 |
| Related to CoCoALib - Design \#1798: Computing in sub polyring |  |  | New | 22 Mar 2024 |

## History

\#1-04 Jan 2022 11:18-John Abbott

- Status changed from New to In Progress
- \% Done changed from 0 to 10

I have modified the code to detect univariate inputs and to handle them "cleverly". The result is noticeably faster!
Now to see what it does with inputs like $\operatorname{gcd}\left(x^{*} y[1],(x+1)^{\star} y[2]\right)$ which obviously involves only $x$ and neither of $y[1]$ and $y[2]$.
NOTE: my sudoku program now takes about 3s for one example where previously it took about 40s!

## \#2-07 Jan 2022 20:23 - John Abbott

- Assignee set to John Abbott
- \% Done changed from 10 to 20

[^0]
## \#3-21 Jan 2022 12:36 - John Abbott

- Related to Feature \#1197: IsZeroDet: new fn added


## \#4-21 Jan 2022 13:07-Anna Maria Bigatti

- Related to Design \#1649: Add file SparsePolyOps-vector.C added


## \#5-16 Mar 2024 21:44 - John Abbott

- Priority changed from Normal to High

This may well be length to resolve properly, but I really should look at it soon.

## \#6 - 19 Mar 2024 20:18 - John Abbott

The problem code is in SparsePolyOps-RingElem.C around line 718 (search for SyzOfGens or maybe just syz).

I think I need a combination of ContentWRT and maybe ContentFreeFactor. Time to look these up in the CoCoALib doc - I do hope I actually wrote some doc for those two fns!

## \#7-19 Mar 2024 22:10-John Abbott

- \% Done changed from 20 to 30

Made some progress. This is more tedious than I thought... the doc for CoCoALib could be better... (and the design too)

## \#8 - 20 Mar 2024 22:03 - John Abbott

- Status changed from In Progress to Resolved
- \% Done changed from 30 to 70

The new code seems to work now, and is faster if the polys are recognized as univariate.
What I do not understand is why syz is so slow:

```
/**/ use ZZ/(19)[u[1..100],x,y[1..100]];
/**/ f1 := x^4 -x^2 +7*x -7;
/**/ f2 := x^4 +4*x^3 -6*x^2 -6*x +7;
/**/ g1 := subst(f1,x,u[1]-y[1]);
/**/ g2 := subst(f2,x,u[1]-y[1]);
/**/ t0 := CpuTime(); syz([g1,g1]); TimeFrom(t0);
--> TAKES 11s on my computer
```

BUT if I do the computation in the smallest suitable ring (with indets $u[1], x, y[1])$ then it takes 0.02 s Probably this should be a new issue!

## \#9-21 Mar 2024 10:01 - Anna Maria Bigatti

John Abbott wrote:

The new code seems to work now, and is faster if the polys are recognized as univariate.
What I do not understand is why syz is so slow:
[...]
BUT if I do the computation in the smallest suitable ring (with indets $u[1], x, y[1]$ ) then it takes 0.02 s

On my computer this takes 0.222 s ( 0.001 with indets $u[1], x, y[1]$ ).
Do you have debugging on?

## \#10-21 Mar 2024 10:25-John Abbott

Ah yes, I do have debugging on.
Do you get a measurable time difference if the ring contains just 3 indets or if it contains 201 indets? Or even 2001? It could be that there is some debugging check which takes a lot of time...

I'll investigate further this evening.

## \#11-21 Mar 2024 11:31-Anna Maria Bigatti

It seems it's the "high number of variables" problem, and syz itself is quite fast: this examples takes $\sim 4 s$ on my computer. Try with verbosity (first check out by verbosity additions):

```
/**/ SetVerbosityLevel(130);
/**/ /**/ use ZZ/(19)[u[1..400],x,y[1..400]];
/**/ f1 := x^4 -x^2 +7*x -7;
/**/ f2 := x^4 +4*x^3 -6*x^2 -6*x +7;
/**/ g1 := subst(f1,x,u[1]-y[1]);
/**/ g2 := subst(f2,x,u[1]-y[1]);
/**/ t0 := CpuTime(); syz([g1,g1]); TimeFrom(t0);
```


## \#12-21 Mar 2024 11:33-Anna Maria Bigatti

- Related to Slug \#1057: Slug: Polynomial ring contructor slow with (big) matrix ordering added


## \#13-21 Mar 2024 11:54-Anna Maria Bigatti

- Related to Slug \#1068: PolyRing constructor: NewOrdvArith computed twice added


## \#14-21 Mar 2024 12:02-Anna Maria Bigatti

The problem for multivariate syz with many indets (from note-8 on) seems to be considered in \#1057 and \#1068, so I would just close this issue.

## \#15-21 Mar 2024 20:18- John Abbott

- Status changed from Resolved to Closed
- \% Done changed from 70 to 100
- Estimated time set to 5.33 h

I have added some new tests to test-SparsePolyRing1.C.
I conform that Anna's example from comment 11 does seems to spend a long time "doing not much (apparently)" then whoosh the GB computation is over in a flash!
The original example is now completed in about 0.001 s (on my Linux laptop).
Closing.

## \#16-22 Mar 2024 09:25 - John Abbott

- Related to Design \#1798: Computing in sub polyring added


[^0]:    I have checked in my first change (so that Anna can experiment with it)
    There is more to come (if/when I find time).

