

CoCoALib - Design #1500

IsDivisible in a field?

05 Oct 2020 14:29 - John Abbott

Status:	Closed	Start date:	05 Oct 2020
Priority:	Normal	Due date:	
Assignee:	John Abbott	% Done:	100%
Category:	Safety	Estimated time:	3.99 hours
Target version:	CoCoALib-0.99800	Spent time:	3.95 hours
Description			
What should IsDivisible(a,b) do with arguments in a field?			
Currently it returns the same as not(b = 0).			
This is mathematically correct, but I have just seen an example of writing a function where IsDivisible was called on elements of QQ... the programmer knew that the value was an integer but had overlooked that it was actually represented as a rational.			
So... should IsDivisible give an error if handed elements of a field? If the user really wants to test not(b=0) then it is surely better to write it explicitly...?			
Related issues:			
Related to CoCoALib - Design #377: IsDivisible -- exact semantics?		Closed	19 Jun 2013
Related to CoCoALib - Design #1085: Fns with "OUT" args: should they give ERR...		Closed	30 Jun 2017

History

#1 - 05 Oct 2020 14:32 - John Abbott

This might be a bit like computing gcd: strictly we can define it for a field, but we opted instead to give an error.

The idea is to help the programmer avoid mistakes. What actually happened was that the discriminant of a polynomial in $\mathbb{Q}\mathbb{Q}[x]$ which has integer coeffs is itself an integer, and then the program had to look for a special non-divisor of this integer... but instead the program entered an infinite loop (even though the code "looked correct").

#2 - 05 Oct 2020 14:33 - John Abbott

- Related to Design #377: IsDivisible -- exact semantics? added

#3 - 08 Oct 2020 12:35 - John Abbott

- Status changed from New to In Progress

- % Done changed from 0 to 10

I am becoming increasingly convinced that it is better to throw an error if the args are in a field (since it makes little sense to test for divisibility in a field).

Another related matter is whether IsDivisible should allow automatic conversion... perhaps that should be another issue?

#4 - 09 Oct 2020 09:19 - John Abbott

- Description updated

This is a bit less clear than I had previously thought. In the file ring.C the function IsDivisible (with 3 args) is used fairly widely: here is an example (around ring.C:700)

```

RingElem div_SameRing(ConstRefRingElem x, ConstRefRingElem y)
{
    const ring& Rx = owner(x);
    CoCoA_ASSERT(Rx == owner(y));
    if (IsZeroDivisor(y)) CoCoA_THROW_ERROR(ERR::DivByZero, "RingElem / RingElem");
    RingElem ans(Rx);
    if (!Rx->myIsDivisible(raw(ans), raw(x), raw(y)))
        CoCoA_THROW_ERROR(ERR::BadQuot, "RingElem / RingElem");
    return ans;
}

```

This would become annoyingly messy if we had to handle fields in a special way.

Perhaps the solution is that myIsDivisible with 3 args should have another name?

#5 - 09 Oct 2020 11:03 - John Abbott

I have tried implementing the change (*i.e.* IsDivisible throws if given args in field).
Two tests fail: test-IsInteger1 and test-OrderedDomain1.

Mmmm. What to do?

#6 - 09 Oct 2020 20:06 - John Abbott

- Assignee set to John Abbott

- % Done changed from 10 to 20

A comment about the code excerpt in comment 4. The call to IsZeroDivisor is superfluous; perhaps it was put there to give a more informative error mesg?

Anyway, it would make more sense to call IsZeroDivisor **after** having established that division fails; certainly for arithmetic in $\mathbb{Z}/(N)$ attempting to divide and testing for being a zero-divisor are largely the same computation, so testing IsZeroDivisor may almost double the computational cost in $\mathbb{Z}/(N)$.

For much the same reason, **it is not worth** having a non-virtual myIsDivisible which calls IsZeroDivisor, and if not then calls a virtual fn to do the actual work.

A possible solution is to give IsDivisible an optional 3rd param *e.g.* **PermitFieldElems**. In other words we offer two versions of IsDivisible: one which gives error when handed field elems, and one which does not. The default would be the version which does signal an error (to protect the unwary user).

#7 - 09 Oct 2020 21:19 - John Abbott

Also ex-RingElem1 fails...

#8 - 12 Oct 2020 20:52 - John Abbott

I still like the idea of a 3rd param (or perhaps two fns with similar names).

My preference is that a call like `IsDivisible(x,y)` gives error if the div-test is in a field; to permit testing also in a field the call should look "a bit more complicated" (but not too much).

An advantage of two different fns is that we do not need to define a new type to use as the 3rd param -- yes, it is effectively a boolean, but a new and specific type would make the code more readable.

If we opt for a 3rd param, there are two possible impls:

- **(A)** there are 2 poss values for the 3rd param (e.g. `DisallowFields` and `AllowFields`)
- **(B)** there is 1 poss value for the 3rd param (e.g. `AllowFields`)

Option **(B)** is perhaps marginally simpler to implement, but might make it slightly trickier if one wants to call a fn passing param saying which sort of div-test to perform [TBH I cannot imagine when one might want to do this]. With option **(A)** one could simply pass whichever of the two values is the desired one. If there are two distinct div-test fns, then the fn to use could be passed as param.

UPDATE: a possible alternative name could be `IsDivisible_AllowFields`; the name is long (probably a good thing), it is also clear (good!).

#9 - 14 Oct 2020 10:59 - John Abbott

Today my preference is for `IsDivisible` and `IsDivisible_AllowFields`.

I think these names are fairly clear, and would also be clear if they needed to be passed as a parameter.

Internal impl would probably use a fn `IsDiv(a,b,bool)` where bool indicates whether fields are allowed -- this design should maximise code sharing.

#10 - 14 Oct 2020 20:21 - John Abbott

- % Done changed from 20 to 50

Oh wow! There are a lot more `IsDivisible` functions than I thought... :-)

SERIOUS QUESTION

What should the following call to `IsDivisible` do?

```
// Assume R1, R2, R3 are different rings: R3 can be promoted to R2 (not a field)
RingElem a(R1);
RingElem b = one(R2);
RingElem c = one(R3);
IsDivisible(a,b,c); // error or not?
```

The point is that **a is in the wrong ring**. Note that `a = b/c`; will succeed (and automatically change the ring of a to be R2).

#11 - 14 Oct 2020 21:21 - John Abbott

- Status changed from *In Progress* to *Feedback*
- % Done changed from 50 to 90

There are now 20 different `IsDivisible` functions (half of them are actually `IsDivisible-AllowFields`).

I have modified `test-IsInteger1`, `test-OrderedDomain1` and `ex-RingElem1` to call `IsDivisible-AllowFields` instead of `IsDivisible`. They all work now.

Regarding the question in comment 10: I have opted to make `IsDivisible(a,b,c)` behave like $a = b/c$ if the division succeeds (otherwise `a` should remain unchanged, but I am not sure I want to guarantee that in the doc).

#12 - 23 Oct 2020 10:57 - John Abbott

- Status changed from *Feedback* to *Closed*
- % Done changed from 90 to 100
- Estimated time set to 3.99 h

#13 - 27 Oct 2020 10:22 - John Abbott

- Related to Design #1085: Fns with "OUT" args: should they give `ERR::MixedRings?` added