

## CoCoALib - Bug #1069

### GroebnerFan: ERROR: Matrix must be invertible

17 May 2017 11:37 - John Abbott

<b>Status:</b>	In Progress	<b>Start date:</b>	17 May 2017
<b>Priority:</b>	Normal	<b>Due date:</b>	
<b>Assignee:</b>		<b>% Done:</b>	10%
<b>Category:</b>	Improving	<b>Estimated time:</b>	0.00 hour
<b>Target version:</b>	CoCoALib-1.0	<b>Spent time:</b>	3.00 hours
<b>Description</b>			
The following causes an error:			
<pre>use QQ[x,y,z]; I := ideal(x^3*y^6 + 2*x^3*y*z^5, x^3*y^4*z^2 + 2*x*y^3*z^2, 2*y^5*z^3 + 2*x^5*z); fan := GroebnerFanIdeals(I); --&gt; ERROR: Matrix must be invertible</pre>			
<b>Related issues:</b>			
Related to CoCoALib - Slug #1049: GroebnerFan: slow examples		<b>In Progress</b>	<b>19 Apr 2017</b>
Related to CoCoA-5 - Design #984: GroebnerFanIdeals: order matrices sometimes...		<b>New</b>	<b>26 Nov 2016</b>

### History

#### #1 - 17 May 2017 11:37 - John Abbott

- Related to Slug #1049: GroebnerFan: slow examples added

#### #2 - 17 May 2017 14:29 - Anna Maria Bigatti

Smaller example, but still too big

```
use QQ[x,y,z];
I := ideal(y^5 - z^5, x^3*y*z - x*z, z^8 - x^5*z);
fan := GroebnerFanIdeals(I); --> ERROR: Matrix must be invertible
```

#### #3 - 17 May 2017 14:29 - Anna Maria Bigatti

- Subject changed from GFAN: ERROR: Matrix must be invertible to GroebnerFan: ERROR: Matrix must be invertible

#### #4 - 17 May 2017 15:09 - John Abbott

Another smaller example (but not as small as Anna's)

```
I= ideal(x^8 + 2*x*y^5*z^2, x*z^7 + 2*y*z, 2*x^4*y^2*z^2 + 2*x*y^2*z);
```

#### #5 - 17 May 2017 15:18 - John Abbott

Here is a guess as to what the problem is... for matrix orderings there was some "funny trick" using a matrix reduced modulo a prime around 32000... this could cause a matrix to appear not to be of full rank.

#### #6 - 17 May 2017 15:39 - Anna Maria Bigatti

John Abbott wrote:

Here is a guess as to what the problem is... for matrix orderings there was some "funny trick" using a matrix reduced modulo a prime around 32000... this could cause a matrix to appear not to be of full rank.

spot on! (As I worked on OrdvArith this morning I knew where to add the check)

```
--> ERROR: Argument to a numerical function too large (value would be too big)
--> [CoCoALib] OrdvArith::MatrixOrderingMod32749Impl ctor
--> WHERE: at line 89 (column 8) of GroebnerFan.cpkg5
```

#### #7 - 17 May 2017 15:45 - John Abbott

- Status changed from New to In Progress

- % Done changed from 0 to 10

The correct solution would be to have a proper implementation of matrix orderings without any "tricks" (too bad if it slow).

An alternative could be to try several different primes, and pick one which works (rather than having the prime "hard-wired" into the code). This should be OK so long as no exponent of an indeterminate becomes bigger than the prime. Nevertheless it would still be a "tricky" solution.

#### #8 - 17 May 2017 15:45 - John Abbott

- Related to Design #984: GroebnerFanIdeals: order matrices sometimes have "large" entries added

#### #9 - 17 May 2017 15:46 - John Abbott

Another approach would be to persuade GFan to produce matrices with smaller entries... this is probably not so easy!

#### #10 - 17 May 2017 15:52 - Anna Maria Bigatti

John Abbott wrote:

Another approach would be to persuade GFan to produce matrices with smaller entries... this is probably not so easy!

It might be MakeNonNeg to make the big entries.

### #11 - 18 May 2017 21:00 - John Abbott

Here are some order matrices with not-too-big entries whose det is a multiple of 32749:

```
mat ([[28, 5, 27],  
[0, 28, 29],  
[19, 27, 1]]);
```

```
mat ([[26, 1, 28],  
[29, 19, 0],  
[5, 27, 29]]);
```

```
mat ([[29, 26, 3],  
[0, 23, 29],  
[23, 2, 28]]);
```

```
mat ([[28, 27, 3],  
[2, 20, 29],  
[25, 0, 29]]);
```

```
mat ([[23, 2, 26],  
[2, 29, 27],  
[27, 29, 4]]);
```

```
mat ([[29, 3, 25],  
[29, 23, 2],  
[6, 29, 29]]);
```

```
mat ([[29, 0, 20],  
[23, 28, 1],  
[0, 27, 26]]);
```

I did try (randomly) searching for some matrices with max entry 28 (or less), but did not find any.

### #12 - 19 May 2017 11:16 - John Abbott

Here is another example which gives trouble:

```

use QQ[x,y,z];
I := ideal(-66*z^5 +205*x*y*z^2, 134*x^3*y*z +355*y, 97*y^4*z +410*z^2);
GF := GroebnerFanIdeals(I);

```

You can probably replace the coeffs with smaller ones.

### #13 - 19 May 2017 11:45 - John Abbott

Here is another "slightly smaller" matrix whose determinant is zero mod 32749:

```

mat([[1,3,35],
     [32,4,3],
     [3,30,4]]);

```

### #14 - 19 May 2017 11:48 - Anna Maria Bigatti

Just to clarify: this means that one cannot create this ring, hmmmmm....

```

/**/ NewPolyRing(QQ, "x,y,z", mat([[1,3,35],
                                  [32,4,3],
                                  [3,30,4]]), 0);

```

### #15 - 19 May 2017 11:55 - Anna Maria Bigatti

I just removed the coefficients to John's last example:

```

use QQ[x,y,z];
I := ideal(z^5 +x*y*z^2, x^3*y*z +y, y^4*z +z^2);
GF := GroebnerFanIdeals(I);

```

This fail because  $\det(M)=0 \pmod p$ .

The first two examples failed because there where entries  $> p$  (now detected, and giving appropriate error message)

**#16 - 17 Dec 2018 13:55 - John Abbott**

Yet another example which fails:

```
use QQ[x,y,z,t];
L := [y^2*z + x*y*t + 1, x^3 + x*y^2, y^3 + z*t^2];
GF := GroebnerFanIdeals(ideal(L));
```

And another...

```
L := [y^2*z + x*z*t + 1, x^3 + x*y^2, z^3 + y*t^2];
```

**#17 - 17 Dec 2018 15:09 - Anna Maria Bigatti**

I get

```
--> ERROR: Argument to a numerical function too large (value would be too big)
--> [Cocoalib] OrdvArith::MatrixOrderingMod32749Impl ctor
```

which is probably as much as we can do.

**#18 - 17 Dec 2018 15:16 - John Abbott**

Indeed, if we accept that an error is appropriate then message is reasonable.

It would be better if CoCoA could handle more general orderings (even if only slowly).

Part of the problem is that GFan tends to produce matrices with unnecessarily large entries (I think).

Maybe we can do an internal "matrix order" impl which uses a larger prime (if the platform is 64-bit)?  
Or even a completely general one (which will surely be quite slow).