# CoCoALib - Slug #1049

## GroebnerFan: slow examples

19 Apr 2017 16:37 - John Abbott

| | | | |
|---|---|---|---|
| **Status:** | In Progress | **Start date:** | 19 Apr 2017 |
| **Priority:** | Low | **Due date:** | |
| **Assignee:** | | **% Done:** | 10% |
| **Category:** | Various | **Estimated time:** | 0.00 hour |
| **Target version:** | CoCoALib-1.0 | **Spent time:** | 2.50 hours |

**Description**

Possibly not too relevant to CoCoA(Lib), but I wanted to collect some challenging GFAN examples.

**Related issues:**

| | | |
|---|---|---|
| Related to CoCoA-5 - Slug #1047: NewPolyRing with user defined ordering is sl... | **Closed** | **18 Apr 2017** |
| Related to CoCoALib - Feature #780: GroebnerFan/ExternalLib-GFan: improve pac... | **In Progress** | **24 Sep 2015** |
| Related to CoCoALib - Slug #1057: Slug: Polynomial ring contructor slow with ... | **In Progress** | **04 May 2017** |
| Related to CoCoALib - Bug #1069: GroebnerFan: ERROR: Matrix must be invertible | **In Progress** | **17 May 2017** |
| Related to CoCoA-5 - Design #984: GroebnerFanIdeals: order matrices sometimes... | **New** | **26 Nov 2016** |

### History

#### #1 - 19 Apr 2017 16:41 - John Abbott

Here are some GroebnerFanIdeals examples which take a long time:

```
use QQ[x,y,z];
I := ideal(y^3*z -2*x^2*z +2*y^2,  y^4 -2*z^4 -x*y*z); --> more than 7200s
I := ideal(x*y*z^2 -y^2*z^2 -z^4 +2*x^3,  -2*z^4 +2*x*y^2 +2*x*y*z +y); --> about 11000s, maybe 625 cones

I := ideal(-2*z^4 +2*x*z^2 +2*z^3,  -2*y^5 -2*x^4*z -2*y^2*z^3); --> > 860s
I := ideal(x^5 +y^5 -y^2*z^2,  2*x^5 +x^3*y*z -2*x*z); -->  >680 cones, long time (>7000s)
```

Some smaller examples:

```
I := ideal(2*x^3 +8*y^3 -y^2*z -6*z^3,  y^3 -5*x*z^2 +6*y*z +4*y); --> 1580s

I :=  ideal(-5*x^2*y -2*y^2*z +7*x*z +3,  5*x^3 -7*y^3 -5*y^2*z -7*x*y); --> 177 cones, 9s
I :=  ideal(2*x^3 +3*y^2*z +3*y -6,  9*x*y^2 +5*y^3 +x*z^2 -7*x^2); --> 129 cones, 170s
I := ideal(-2*y^3 -z^3 +4*z^2,  x*y*z +8*z^3 +7*x); --> 73 cones
I := ideal(4*x^3 -3*y^2*z +2*y,  6*x^2*y +5*y^3 +z^2); --> 76 cones
I := ideal(-6*y^2*z +9*z^3 +6*x^2,  -9*y^3 +8*x*z^2 +8*z); --> 76 cones
```

#### #2 - 19 Apr 2017 18:43 - Anna Maria Bigatti

*- Related to Slug #1047: NewPolyRing with user defined ordering is slower than CoCoALib added*

#### #3 - 19 Apr 2017 18:44 - Anna Maria Bigatti

*- Related to Feature #780: GroebnerFan/ExternalLib-GFan: improve package added*

#### #4 - 19 Apr 2017 20:56 - John Abbott

The following example fails:

```
GroebnerFanIdeals(ideal(x*y,y^3+x,y^3));
--> ERROR: MinBy: list is empty
--> WHERE: at line 283 (column 18) of list.cpkg5
-->   if L = [] then error("MinBy: list is empty"); endif;
-->                  ^^^^^^^^^^^^^^^^^^^^^^^^^^^^
CONTEXT: function MinBy at line 283 of list.cpkg5
 CALLED BY: function OutEdge at line 165 of GroebnerFan.cpkg5
  CALLED BY: function recursive at line 182 of GroebnerFan.cpkg5
   CALLED BY: function GroebnerFanIdeals at line 228 of GroebnerFan.cpkg5
```

NOTE: this really ought to be a new issue... but hopefully Anna will fix it in a flash!

Another example:

```
GroebnerFanIdeals(ideal(y^2+y*z+1,y*z+z^2));
```

#### #5 - 20 Apr 2017 10:04 - Anna Maria Bigatti

John Abbott wrote:

> NOTE: this really ought to be a new issue... but hopefully Anna will fix it in a flash!

Embarassing error in ReducedGBasis: I'll investigate.
(continuing this on #780)

#### #6 - 21 Apr 2017 11:05 - Anna Maria Bigatti

*- Subject changed from GFAN: slow examples to GroebnerFan: slow examples*

#### #7 - 26 Apr 2017 15:31 - John Abbott

*- Status changed from New to In Progress*

*- % Done changed from 0 to 10*

Here is a strange example:

```
use QQ[x,y,z];
I := ideal(-2*x*y^2 +y^3 +3*y^2*z -z,  2*y*z^2 +3*x^2 +x*z -2*x,  -2*x^2*z +x*z^2 +z^3 +3*z^2);
GFAN := GroebnerFanIdeals(I);
```

It computes 41 ideals then seems to get "stuck" for a long time (but does proceed slowly, using lots of RAM).

**UPDATE** it finished after about 13400s, and the fan contains 204 cones.

#### #8 - 26 Apr 2017 18:00 - John Abbott

Here are some examples with binomial ideals:

```
use QQ[x,y,z];
I := ideal(-4*x*y+10, 9*y^3+4*z^3, -8*x^3-2*y*z^2); --> 65 cones, 0.4s
I :=  ideal(-3*x^3 +4*y^3,  -3*x*y*z +4*y,  -3*x*y^2 -4*z^3); --> 67 cones, 0.4s
I :=  ideal(-2*x^3 -3*y*z^2,  -4*y^3 -3*z^3,  -5*x^2*y +3); --> 67 cones, 0.4s
I :=  ideal(5*x^2*z -5,  -3*z^3 +5*x*y,  y^3 -3*x^2); --> 67 cones 0.4s
```

I do not have much "feel" for Groebner fans, but it did surprise me to find that even a binomial ideal can have such a large Gfan.

**UPDATE**
I ran a longer search and the computer produced these examples (1 trinomial and 2 binomials of degree 3)

```
Fan size: 473
3 ideals with big GFAN:
[
  ideal(65*y^2*z +81*z^2 +23,  63*x^3 +69*y^3,  78*x^2*y +21*z^3),
  ideal(30*x^2*z +91*z^2 +10,  8*x*y^2 +48*z^3,  41*x^3 +40*y^3),
  ideal(76*x*y^2 +42*x^2 +47,  53*x^3 +41*y*z^2,  91*y^3 +63*z^3)
]
Times taken were ["13.496",  "13.946",  "14.488"]
Slowest took 138.48
Slowest example: ideal(73*x*y^2 +53*z^3 +77*x*y,  90*x^3 +44*y*z,  6*y^3 +89*x)
```

As always, I suspect that the coeffs can be replaced by smaller ones.

#### #9 - 15 May 2017 10:40 - John Abbott

Here are some binomial examples of deg 4:

```
Fan size: 182
3 ideals with big GFAN:
[
  ideal(-34*z^4 -80*x*y,  52*y^4 -32*x^3,  -88*x^3*z +22),
  ideal(20*z^4 +43*y^3,  -33*x^4 +80*y*z,  -88*x*y^3 +81),
  ideal(8*y^4 +26*x*z,  39*x^4 +39*z^3,  -40*y*z^3 -98)
]
Times taken were ["1.2748",  "1.2624",  "1.2772"]
```

Probably the coeffs can be replaced by much smaller values (so long as the supports remain the same).

#### #10 - 15 May 2017 12:28 - John Abbott

Here are two deg 5 binomial examples:

```
Fan size: 338
1 ideals with big GFAN:
[
  ideal(60*x^2*y*z -96*x^2,  69*x^3*y^2 +52*z^5,  35*x^5 +3*x*y^4)
]
Times taken were ["2.9786"]

Fan size: 336
1 ideals with big GFAN:
[
  ideal(860*x^2*z^3 -453*x*z^2,  758*y^5 -356*x*z^4,  640*x^5 +802*y^2*z^3)
]
Times taken were ["3.1647"]
```

Again I believe only the supports are important; the coeffs can almost certainly be replaced by smaller ones.

An example in deg=7:

```
Fan size: 696
1 ideals with big GFAN:
[
  ideal(644*x^4*y^3 -519*x*z^6,  -260*y^7 -553*x^4*z,  117*x^2*y*z^2 +24*x*y*z)
]
Times taken were ["7.1751"]
```

Here is a silly example (deg=9):

```
time 23.682  ideal(13*x^9 -88*z^4,  -97*x^4*y^4*z -41*x^4*y^3,  81*x^2*z^7 +10*y^5)
```

```
GFAN size 1826
```

#### #11 - 16 May 2017 15:25 - Anna Maria Bigatti

*- Related to Slug #1057: Slug: Polynomial ring contructor slow with (big) matrix ordering added*

#### #12 - 17 May 2017 11:37 - John Abbott

*- Related to Bug #1069: GroebnerFan: ERROR: Matrix must be invertible added*

#### #13 - 19 May 2017 15:55 - John Abbott

```
Fan size: 634
1 ideals with big GFAN:
[
  ideal(80*x^3 +9*x*y^2 +10,  49*x^3 +32*y^2*z +36*z^2,  45*y^3 +85*z^3 +41)
]
Times taken were ["44.498"]
```

#### #14 - 23 May 2017 11:32 - John Abbott

Having 4 indets makes it easy to find "big" examples:

```
use QQ[x,y,z,t];
I := ideal(x*z^2+x*y+z, z^3+y^2*t, x^2*y+z*t^2, y*z^2+x); --> Time 800s, UnivDenom = 2*10^13090, NumCones = 61
94
I := ideal(y^2*z +t^3 +z*t,  z*t^2 +z,  x*z^2 +y*z*t,  y*z^2 +x^2); -->  Time 95200s, UniDenom = 1.4*10^19, Nu
mCones = 820, MaxDenom = 1280
I := ideal(x^2*z +y*z +t,  y^3 +x,  x^3 +z^3,  x^2*t +t^3); --> ERROR in ctor for MatrixOrdering after 158000s
 (about 44hrs)
I := ideal(x*y*z +y*z*t +y,  z^3 +x^2,  y^2*z +t^3,  x^3 +y^3); --> time 11800s, UnivDenom=2.5*10^379530, NumC
ones=36928
GF := GroebnerFanIdeals(I);
len(GF);
```

Even in deg 2 there are some hard examples:

```
I :=  ideal(x^2 +x*y +y*t,  t^2 +z,  z^2 +1,  y^2 +t); --> UnivDenom = 2.7006*10^114, NumCones = 179, time = 7
.7s
I := ideal(x^2+x*z+z*t, y^2+t, z^2+y, t^2+1); --> UnivDenom = 2*10^104, time = 8.9s, NumCones = 198
I := ideal(z^2+y*t+x, t^2+y, x^2+z*t, y^2+1); --> UnivDenom = 8*10^80, NumCones = 232
```

#### #15 - 11 Mar 2024 11:12 - Anna Maria Bigatti

```
I :=  ideal(-5*x^2*y -2*y^2*z +7*x*z +3,  5*x^3 -7*y^3 -5*y^2*z -7*x*y); --> 177 cones, 9s
t0 := CpuTime(); GFAN := GroebnerFanIdeals(I); TimeFrom(t0);
```

It is 20s on my computer.  Maybe for a change I made in reduction/sugar?

#### #16 - 11 Mar 2024 11:15 - John Abbott

*- Related to Design #984: GroebnerFanIdeals: order matrices sometimes have "large" entries added*