"My Little Project with CoCoALib"

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February 2017, Kassel

- an algorithm in a paper
- a mathematical conjecture
- a new idea
- .. or just homework ;-)

Example 1

"My project" is quite a difficult example:

Computing primary decomposition of 0-dimensional ideals

- Already implemented it in CoCoA-5 (PhD thesis E. Palezzato)
- My project today it to translate it into CoCoALib.

Bibliography:
M.Kreuzer-L.Robbiano book: *Computational Linear and Commutative Algebra*
E.Palezzato PhD Thesis: *Minimal Polynomial, Sectional Matrix, and Applications*
Paper Abb+Big+Palezzato+Robbiano: *Computing and Using Minimal Polynomials*

First steps on the project

1. Sketch on **PAPER**!
2. Compute easy examples by hand, following the algorithm
3. Compute harder example by "reverse engineering"
4. Identify the important objects and steps

Example 2

1. Difficult phase: for my project already done 😃

2. Given $I = \langle x^2 - 4, y - 1 \rangle$. Choose $f = x + y$.
   Let $\mu(z) = MinPolyQuot(f, I) = z^2 - 2z - 3$
   (*i.e.* $\mu(f) \in I$ of min deg) and factorize it $(z + 1)(z - 3)$.
   Let $Q_1 = I + \langle f+1 \rangle$ and $Q_2 = I + \langle f-3 \rangle$: then $I = Q_1 \cap Q_2$

3. $Q_1 = \langle x, y - 1 \rangle^2$, $Q_2 = \langle x - 3, y^2 + 1 \rangle$, $Q_3 = \langle x, y \rangle$. Let
   $I = Q_1 \cap Q_2 \cap Q_3$!!

4. input `ideal`, output `vector<ideal>`.
   Choosing a polynomial, `MinPolyQuot`, `factor`,...

First steps on the computer

1. Start by a meaningful example "step-by-step"

2. **Understand** which **types** you need:
   BigInt, BigRat, ring, RingElem, ideal, list/vector, ..
   (You probably do not need to make a new class)

3. **Understand** which **functions** you need

4. Make a **simple prototype** (in CoCoA-5 or CoCoALib)

Example 3

1. Meaningful example "step-by-step"
   (*ex-PrimaryDecomposition0Dim*)

2. ring, RingElem, ideal, `vector<RingElem>`,
   `factorization<RingElem>`, `RingHom`, ...

3. `MinPolyQuot`, `factor`, ...

4. (there is an excellent prototype in CoCoA-5...)

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On paper work **"top-down"**:
start with the big picture
and identify the "logical blocks"

On computer work **"bottom-up"**:
Make the basic functions (the "logical blocks")
and test each of them

Print, print print!! (Use "verbosity")

Example 4

- PrimaryDecomposition (I don't complete this today)
    - IsPrimary0Dim (*maximal.cpkg5*)
        - RndLinForm

(*ex-IsPrimary0dim*)

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On computer work **"bottom-up"**:
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