# Origami and Trisection of an angle

Department of Mathematics University of Genova

June 7, 2013

## Origami

Huzita's origami axioms are described in terms of the following fold operations:

1. Given two points P and Q, we can make a fold along the crease passing through them.
2. Given two points P and Q, we can make a fold to bring one of the points onto the other.
3. Given two lines m and n, we can make a fold to superpose the two lines.
4. Given a point P and a line m, we can make a fold along the crease that is perpendicular to m and passes through P .
5. Given two points P and Q and a line m, either we can make a fold along the crease that passes through Q, such that the fold superposes P onto m, or we can determine that the fold is impossible.
6. Given two points P and Q and two lines m and n, either we can make a fold along the crease, such that the fold superposes P and m, and Q and n, simultaneously, or we can determine that the fold is impossible.

### Axiom 1

```
Define Axiom1(P,Q)
 If P = Q Then Error("P = Q") EndIf;
 Return (y-P[2])/(Q[2]-P[2])-(x-P[1])/(Q[1]-P[1]);
EndDefine; -- Axiom1
```

### Axiom 3

```
Define Axiom3(m,n)
 phi := CoeffEmbeddingHom(RingOf(x));
 If monic(m)=monic(n) Then Error("Bad Input") EndIf;
 Coefm := apply(phi, Coefficients(m,[x,y,1]));
 Coefn := apply(phi, Coefficients(n,[x,y,1]));
 r := monic((Coefm[1]+Coefn[1])*x+(Coefm[2]+Coefn[2])
                                 *y+(Coefm[3]+Coefn[3]));
 s := monic((Coefm[1]-Coefn[1])*x+(Coefm[2]-Coefn[2])
                                 *y+(Coefm[3]-Coefn[3]));
 Return [r,s];
EndDefine; -- Axiom3
```

### Axiom 5

```
Define Axiom5(P,Q,m)
 R_2 := (Q[1]-P[1])^2+(Q[2]-P[2])^2;
 Circ := (x-Q[1])^2 + (y-Q[2])^2 - R_2;
 I := [Circ,m];
 RGB := ReducedGBasis(Ideal(BringIn(I)));
 ...
 EndIf;
EndDefine; -- Axiom5
```

### Axiom 2

```
Define Axiom2(P,Q)
 If P = Q Then Error("P = Q") EndIf;
 Asse := monic((x-P[1])^2+(y-P[2])^2-
 (x-Q[1])^2-(y-Q[2])^2);
 Return Asse;
EndDefine; -- Axiom2
```

### Axiom 4

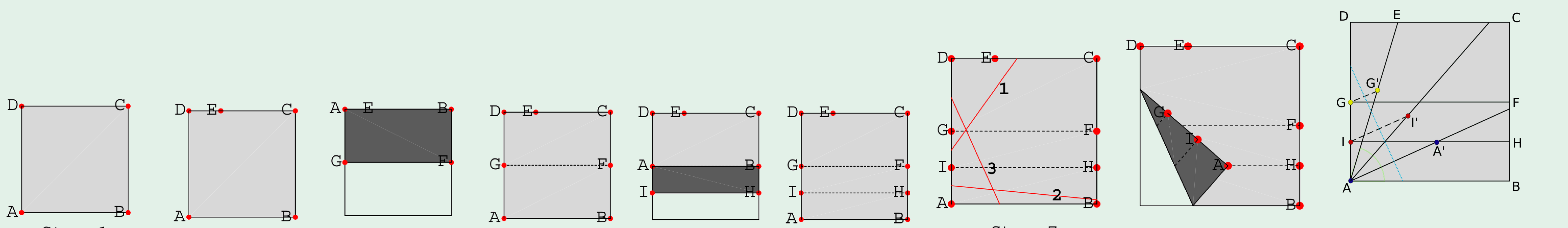
```
Define Axiom4(m,P)
 phi := CoeffEmbeddingHom(RingOf(x));
 Return (y-P[2]) - (phi(CoeffOfTerm(m,y)/
             CoeffOfTerm(m,x))*(x-P[1]));
EndDefine; -- Axiom4
```

### Axiom 6

```
Define Axiom6(P,n,Q,m)
 L := [];
 append(Ref L, Eval(m, [s,t]));
 append(Ref L, Eval(n, [u,v]));
 Axis := (x-Q[1])^2+(y-Q[2])^2-(x-s)^2-(y-t)^2;
 ...
 fold := gens( Elim(s..v, Ideal(K)));
 Return fold;
EndDefine; -- Axiom6
```

## Trisection of an Angle

1. *Steps 1 and 2:* First, we define a square origami paper, whose corners are des- ignated by the points A, B, C and D. The size may be arbitrary, but for our example, let us fix it to 100 by 100. The new origami figure is created with two differently colored surfaces: a light-gray front and a dark-gray back. We then introduce an arbitrary point, say E at the coordinate (30, 100), assum- ing A is at (0, 0).
2. *Steps 3 and 4:* We make a fold to bring point A to point D, to obtain the perpendicular bisector of segment AD. This is the application of (O2). The points F and G are automatically generated by the system. We unfold the origami and obtain the crease F G.
3. *Steps 5 and 6:* Likewise we obtain the crease IH.
4. *Steps 7 and 8:* Step 7 is the crucial step of the construction. We will superpose point G and the line that is the extension of the segment AE, and to superpose point A and the line that is the extension of the segment IH, simultaneously. This is possible by (O6) and is realized by the call of function FoldBrBr. There are three candidate fold lines to make these superpositions possible. The system responds with the query of "Specify the line number"together with the fold lines on the origami image. We reply with the call of FoldBrBr with the additional parameter 3, which tells the system that we choose the line number 3. This is the fold line that we are primarily interested in. However, the other two fold lines are also solutions (which trisect different angles).
5. *Steps 9 and 10:* We will duplicate the points A and I on the other face that is below the face that A and I are on, and unfold the origami. The duplicated points appear as L and J for A and I, respectively. These names are automatically generated. Finally, we see that the segments AJ and AL trisect the angle EAB.




Step 1 Step 2 Step 3 Step 4 Step 5 Step 6 Step 7 Step 8