8th June 2008

Exercise 1 *Consider the homogeneous coordinate ring of the "twisted cubic":*

$$R = K[s^3, s^2t, st^2, t^3]$$

1. *Prove that* $R = P/I$ *where* $P = K[x_0, \dots, x_3]$ *and* $I = I_2\begin{pmatrix} x_0 & x_1 & x_2 \\ x_1 & x_2 & x_3 \end{pmatrix}$
2. *Prove that* R *is CM*
3. *Compute* $\mathrm{HF}_R(j)$, $\mathrm{reg} - \mathrm{index}(R)$
4. *Compare* $\mathrm{reg}(I)$ *and* $\mathrm{reg}(LT_\tau(I))$ *with* τ *any term ordering*

Exercise 2 *Consider the homogeneous coordinate ring of the smooth rational quartic in* $\mathbb{P}^3$

$$R = K[s^4, s^3t, st^3, t^4]$$

1. *Prove that* $R \simeq P/I$ *where* $P = K[x_0, \dots, x_3]$ *and* $I = I_2\begin{pmatrix} x_0 & x_1^2 & x_1x_3 & x_2 \\ x_1 & x_0x_2 & x_2^2 & x_3 \end{pmatrix}$
2. *Prove that* R *is not CM*
3. *Compute* $\mathrm{reg}(I)$

Exercise 3 (Trung) *Let* $\mathcal{C} \subseteq \mathbb{P}^3$ *be a curve whose homogeneous coordinate ring is*

$$R = K[t^{\alpha+\beta}, t^\alpha s^\beta, t^\beta s^\alpha, s^{\alpha+\beta}] \quad \text{with } \alpha > \beta,\ \mathrm{GCD}(\alpha, \beta) = 1$$

and consider $I = I_{\mathcal{C}} \subseteq K[x_0, \dots, x_3]$. *Then* $\mathrm{reg}(I_{\mathcal{C}}) = \alpha$. *Check it in some examples. What about the Cohen-Macaulyness of* R*?*

Exercise 4 *Compute the Betti diagram of 11 randomly chosen points in* $\mathbb{P}^7$. *Compute regularity index (*`RegularityIndex`*) and regularity.*

Exercise 5 *Let* $P = K[x_1, \dots, x_n]$ *and* $F_1, F_2, F_3 \in P$ *homogeneous polynomials which form a regular sequence.*

1. *Assume* $d_i = \deg(F_i)$ *and compute* $\operatorname{reg}(I)$ *where* $I = (F_1, F_2, F_3)$
2. *Can you compute the value of* $\operatorname{reg}(I)$ *where* I *is generated by a regular sequence of degrees* $d_1, \ldots, d_r$ *?*

Exercise 6 *Describe Hilbert function, Hilbert polynomial, Betti diagram, regularity of each possible configuration of 4 distinct points in* $\mathbb{P}^2$.