

Tutorial 2

Betti numbers and generic ideals

Swapping indeterminates:

```
Use Q[x,y,z];
F := x - y^2 - z^3; F;
-z^3 - y^2 + x
-----
Subst(F, [[x,y], [y,x]]);
-z^3 - x^2 + y
-----
I := 1; J := NumIndets();
Subst(F, [[Indet(I),Indet(J)], [Indet(J),Indet(I)]]);
-x^3 - y^2 + z
-----
```

More “mathematical” implementation of change of coordinates:

```
Image(F, RMap(x-y+2z, 3y-z, z));
-z^3 - 9y^2 + 6yz - z^2 + x - y + 2z
-----
```

Generic change of coordinates

```
Define RandMat(R,C, A,B) -- a RxC matrix with entries in A..B
Return Mat([ [Rand(A,B) | J In 1..C] | I In 1..R]);
EndDefine;
RandMat(NumIndets(), NumIndets(), -10, 10);
Mat([
  [-2, -4, 3],
  [3, 0, 9],
  [-4, 3, 6]
])
-----
M := It; -- "It" is the last value
L := RowMat(Indets()) * M; L;
Mat([
  [-2x + 3y - 4z, -4x + 3z, 3x + 9y + 6z]
])
-----
Image(F, RMap(L[1]));
-27x^3 - 243x^2y - 162x^2z - 729xy^2 - 972xyz - 324xz^2 - ....
```

To create strongly stable ideals (Borel fixed)

```
StronglyStableIdeal([x^3y^2, y^6]);
Ideal(x^2y^4, xy^5, y^6, x^5, x^4y, x^3y^2)
-----
IsStronglyStable(It); -- ... of course ...
TRUE
-----
```

Change ring, change ordering

```
Use PLex := Q[x,y,z], Lex;
F := x - y^2 - z^3; F;
x - y^2 - z^3
-----
```

```
Use PDegRevLex := Q[x,y,z], DegRevLex; -- default value
BringIn(F);
-z^3 - y^2 + x
-----
```

```
Use PDegLex := Q[x,y,z], DegLex;
BringIn(F);
-z^3 - y^2 + x
-----
```

Compute gin

```
L := [y^3z, xz^2];
Gin(Ideal(L)); -- 5 computations with small random coeffs
Gin5(Ideal(L)); -- 2 computations with big random coeffs
Use PDegLex;
Gin(Ideal(BringIn(L)));
Use PDegRevLex;
Gin(Ideal(BringIn(L)));
```