9th June 2008

Exercise 1 *Let* $L \in P_1$ *be a linear form. Can you find examples with* $\operatorname{reg}(I + (L)) > \operatorname{reg}(I)$ *?*

Problem (Caviglia) *Is* $\operatorname{reg}(I + (L))$ *bounded by a polynomial function (possibly quadratic) of* $\operatorname{reg}(I)$ *?*

Exercise 2 *Compare the regularities of the ideals* I *and* J *with those of* $\sqrt{I}$ *,* I^2 *,* $I + J$ *,* IJ *in some examples.*

Problem *Compare* $\operatorname{reg}(\sqrt{I})$ *and* $\operatorname{reg}(I)$ *with* I *is a monomial ideal. Have you some guess in* $P = K[x, y]$ *? In general?*

(The answer is known, (Ravi))

Exercise 3 (Chardin-D'Cruz) *Let* R *be the homogeneous coordinate ring of the monomial surface* $S \subseteq \mathbf{P^5}$ *parametrized by*

$$(a^{13}, ab^{12}, a^5c^8, a^5bc^7, a^7b^5c, b^9c^4)$$

Let $I = I(S) \subseteq k[X_0, \ldots, X_5]$ *and* J *the ideal generated by the polynomials of* I *of degree* ≤ 21 *. By using CoCoA verify that*

1. $J = I \cap (X_1, \ldots, X_5)$
2. $\operatorname{reg}(I) = 32$ *and* $\operatorname{reg}(J) = 24$
3. *depth*$P/I = 1$ *and depth*$P/J = 2$.

Exercise 4 *Let* n, m *be positive integers and let*

$$I_{m,n} = (x^m t - y^m z, z^{n+2} - xt^{n+1}) \subseteq K[x, y, z, t]$$

By CocoA's help, check (in particular cases) the following equalities

1. $\operatorname{reg}(I_{m,n}) = m + n + 2$ *(complete intersection)*
2. $\operatorname{reg}(\sqrt{I_{m,n}}) = m \cdot n + 2$